Abstracts of Talks

35th Annual Workshop in Geometric Topology
Calvin College
June 14–16, 2018

Invariants of Knots and Links

András Stipsicz
Alfréd Rényi Institute of Mathematics
Budapest, Hungary

Lecture 1: Invariants of knots and links
We plan to review definitions of the Alexander polynomial using Kauffman states and grid diagrams. These approaches then lead to knot Floer homology; we sketch the definition of these groups.

Lecture 2: Knot Floer homology
We define knot Floer groups through grid diagrams, verify their main properties and apply these tools in solving some geometric problems, including the Milnor conjecture for torus knots.

Lecture 3: The Upsilon function of knots
Applying the appropriate version of knot Floer homology and some ideas from homological algebra, a function-valued knot invariant can be derived. This function can be conveniently used in the study of the smooth concordance group. We show some illustrative examples of such results.

Fixed Points Results for Multivalued Contractive Mappings with CLR Property

Afrah Ahmad Noman Abdou
Jeddah University

Abstract. In this work, we define a new properties which contains the property (CLR) and (owc) for four single and multi-valued maps and give some new common fixed point results. Our results extend and improve some results given by some authors.

Matrix Transformation of Generalized Shifted Chebyshev Koornwinder's Type Polynomials Basis

Mohammad A. AlQudah *German Jordanian University*

Abstract. This paper provides an explicit form of the generalized shifted Chebyshev Koornwinder's type polynomial of first kind in terms of Bernstein basis of fixed degree n. Moreover, explicit forms of generalized shifted Chebyshev Koornwinder's type and Bernstein polynomials bases transformations were provided. Joint work with Maalee N. AlMheidat.

Slender Properties of HNN Extensions

Michael Andersen
Brigham Young University

Abstract. We hope to show that ascending HNN extensions of cm-slender groups are also cm-slender.

Monotone Maps of the Pontryagin Sphere to Itself

Robert J. Daverman
University of Tennessee

Abstract. The talk will report on both a theorem and an example. The theorem shows that degree one monotone maps in this category are near-homeomorphisms and (2) the example reveals that the degree one hypothesis is necessary. This is based on joint work with Tom Thickstun.

Unoriented Cobordism Maps on Link Floer Homology

Haofei Fan
UCLA

Abstract. We study the problem of defining maps on link Floer homology induced by unoriented link cobordisms. We provide a natural notion of link cobordism, disoriented link cobordism, which tracks the motion index zero and index three critical points. Then we construct a map on unoriented link Floer homology associated to a disoriented link cobordism. Furthermore, we will discuss some potential applications on the involutive upsilon invariants and unoriented four-ball genus.

Complexity of Virtual Multistrings

David Freund
Dartmouth College

Abstract. A virtual n-string α is a collection of n closed curves on an oriented surface M. Associated to α, there are two natural measures of complexity: the genus of M and the number of intersection points. By considering virtual n-strings up to equivalence by virtual homotopy, i.e., homotopies of the component curves and stabilizations/destabilizations of the surface, a natural question is whether these quantities can be minimized simultaneously. We show that this is possible for non-parallel virtual n-strings and that, moreover, such a representative can be obtained by monotonically decreasing genus and the number of intersections from any initial representative.

Stratified Spaces and Intersection Homology

Greg Friedman
Texas Christian University

Abstract. Intersection homology was developed by Goresky and MacPherson to extend Poincare duality and related invariants from manifold theory to "singular spaces," such as singular algebraic varieties. We'll provide an introduction to these topics.

The Family of Induced Representations for Quantum Groups at a Primitive 4th Root of Unity

Matthew Harper
The Ohio State University

Abstract. For any non-zero complex number, we can construct an induced representation of $U_\xi(\mathfrak{sl}_2)$ for ξ a root of unity. As shown by Ohtsuki, these representations can be used to define a knot invariant, in this case it is Alexander polynomial. From the rank one case, we build the analogous rank two invariant and examine the tensor product structure of these representations, and the resulting "$\mathfrak{sl}_3$" skein relation.

New Symmetries of Stable Homotopy Groups

Mohammad Kang
Wayne State University

Abstract. This will really be a talk about abstract algebra and some very elementary number theory, but the motivation for the work comes from topology. Work of Bousfield, Ravenel, Morava, Miller, Wilson, and others during

the 1970s and 1980s established that the stable homotopy groups of spheres can be decomposed into many periodic families, each of which repeats every $2(p^n - 1)$ dimensions, where p is a prime and n is a nonnegative integer; and furthermore, if one fixes p and n, then there are spectral sequences that one can, in principle, use to calculate the $2(p^n - 1)$-periodic families in the stable homotopy groups of spheres. These spectral sequence calculations are extremely difficult, however, and complete calculations have only been made for $n < 3$.
When $p > n+1$, these spectral sequence calculations begin with the cohomology of a certain differential graded algebra defined by Ravenel. In this talk, we give a very explicit description of this (surprisingly simple!) differential graded algebra, and we explore the problem of finding symmetries (i.e., automorphisms) of this differential graded algebra for large n. We show how we reduce the problem of finding such symmetries to a very explicit and elementary problem in number theory, and we demonstrate some new symmetries–which give rise to new operations on the $2(p^n - 1)$-periodic stable homotopy groups of certain CW-complexes called Smith-Toda complexes–which our elementary, explicit approach produces. (The focus of the talk will be on the very approachable algebra and elementary number theory which we used to get our new results; the ideas from homotopy theory are there to motivate our work, but our talk should be understandable to an audience which does not know any homotopy theory!)

Lifting Branched Covers to Braided Embeddings

Sudipta Kolay
Georgia Tech

Abstract. An embedding of a manifold M^k in a trivial disc bundle over N^k is called braided if projection onto the first factor gives a branched cover. This notion generalizes closed braids in the solid torus, and gives an explicit way to construct many embeddings in higher dimensions.
One could ask which branched covers lift to braided embeddings. This question has been well studied for honest covering maps by Hansen and Petersen.

In this talk, we will discuss about this question for branched covers over low (i.e. less than 5) dimensional spheres.

The Width of Satellite Knots

Zhenkun Li
MIT

Abstract. Width is a knot invariant first introduced by Gabai in 1980s. In the paper, I will discuss the relationship between width of a satellite knots and its companion. In particular, as a joint work of me and Qilong Guo, we proved that $w(K)$ is no less than $n^2w(J)$, where $w(.)$ is the width of a knot and K is a satellite knot with companion J and winding number n. Furthermore, recently we also proved that in the case that K is a whitehead double of J, where the winding number is 0, we can replace n^2 by 4, which is the square of the wrapping number of K. The general case is still open though.

Relatively Hyperbolic Groups have Semistable Fundamental Group at Infinity

Mike Mihalik
Vanderbilt University

Abstract. Suppose G is a 1-ended finitely generated group that is hyperbolic relative to $\mathbf{P}$ a finite collection of 1-ended finitely generated proper subgroups of G. Our main theorem states that if the boundary $\partial(G, \mathbf{P})$ has no cut point, then G has semistable fundamental group at ∞. Under mild conditions on G and the members of $\mathbf{P}$ the 1-ended hypotheses and the no cut point condition

can be eliminated to obtain the same semistability conclusion. We give an example that shows our main result is somewhat optimal. Finally, we improve a "double dagger" result of F. Dahmani and D. Groves.

Geometric Groupoid Models for C^* Algebras

Atish Mitra
Montana Tech

Abstract. We use ideas from geometric topology to construct groupoid models for certain C^* algebras. The distinctive features of our method is that our groupoid modeling is functorial, and easy to visualize geometrically.
This is joint work with K. Austin.

Boundaries of Generalized Baumslag-Solitar Groups

Molly Moran
Colorado College

Abstract. Bestvina defined a Z-structure on a group G to generalize the theory of boundaries of CAT(0) and hyperbolic groups. In this talk, we will add to the known collection of groups admitting Z-structures by showing that all generalized Baumslag-Solitar groups admit Z-structures.
Joint work with Craig Guilbault (University of Wisconsin-Milwaukee) and Carrie Tirel (University of Wisconsin-Fox Valley)

Growth Series of CAT(0) Cubical Complexes

Boris Okun
University of Wisconsin Milwaukee

Abstract. Let X be a CAT(0) cubical complex. If X is cocompact, the growth series of X at x, $G_x(t) = \sum_{y \in X} t^{d(x,y)}$, is a rational function of t. In the case when X is the Davis complex of a right-angled Coxeter group it is well-known that $G_x(t) = 1/f_L(-t/(1+t))$, where f_L denotes the f-polynomial of the link L of a vertex of X. We obtain a similar formula for general cocompact X. We also obtain a simple relation between the growth series of individual orbits and the f-polynomials of various links. In particular, we get a simple proof of reciprocity of these series ($G_x(t) = \pm G_x(t^{-1})$) for an Eulerian manifold X. This a joint work with Rick Scott (Santa Clara University).

Computing the Smooth Nonorientable Four Genus of the Negative Nine Two Knot

Jacob Pichelmeyer
Kansas State University

Abstract. The smooth nonorientable four genus of a knot K was defined by Murakami and Yasuhara in 2000 to be the minimal first Betti number of any non-orientable surface smoothly and properly embedded in D^4 with boundary K. Until recently, this invariant was unknown for many of the prime knots in the standard Rolfson knot table. However, in 2018, Stanislav Jabuka and Tynan Kelly computed the smooth nonorientable four genus for all prime knots in the Rolfson knot table with 8 or 9 crossings. We will consider a particular 9-crossing knot, -9_2, and walk through Jabuka and Kelly's computation of it's nonorientable four genus.

Harmonic Spinors on the Davis Hyperbolic 4-manifold

John Ratcliffe
Vanderbilt University

Abstract. Harmonic spinors are solutions of the Dirac operator. We use the G-spin theorem to show that the Davis hyperbolic 4-manifold admits non-zero harmonic spinors. This the first example of a closed hyperbolic n-manifold with $n > 2$ that admits non-zero harmonic spinors.

Simplicial Inverse Systems and Extension Theory

Leonard R. Rubin
University of Oklahoma

Abstract. An inverse system is called *simplicial* if all its coordinate spaces are triangulated polyhedra and all its bonding maps are simplicial with respect to these triangulations. It has been shown by S. Mardešić that if a compact metrizable space X has $\dim X \geq 1$, and X is the inverse limit of a simplicial inverse sequence of compact triangulated polyhedra, then X must contain an arc. It follows then that a pseudo-arc cannot be the limit of such an inverse sequence. The first author has studied simplicial inverse systems, and we shall report on some outcomes of that research.
The motivation for studying simplicial inverse systems derives from joint research of L. Rubin and V. Tonić which is still in its preliminary stages. In extension theory, it is typically valuable to be able to represent a given metrizable compactum as the limit of a simplicial inverse sequence of compact triangulated polyhedra. But, as noted above, this is not always possible. Our program involves finding a "suitable replacement" Z for a given metrizable compactum X in such a manner that all the extension theoretic properties of X exist for Z. This means that if a given CW-complex is an absolute extensor for X, then it is also an absolute extensor for Z. Our plan is to find such a Z that is the limit of a simplicial inverse sequence of compact triangulated polyhedra and such that X is the cell-like image of Z under a map that is induced by the inverse sequence representing Z.
Joint work with Vera Tonić, University of Rijeka.

Action Dimension of Some Simple Complexes of Groups

Kevin Schreve
University of Michigan

Abstract. The geometric dimension of a discrete group G is the minimal dimension of a model for the classifying space BG. The action dimension of G is the minimal dimension of a manifold model. I will talk about some computations of the action dimensions for certain complexes of groups and fundamental groups of complex hyperplane complements. This is joint work with Michael Davis and Giang Le.

Deficient and Multiple Points of Maps into Manifolds

Stanisław Spież
Institute of Mathematics, Polish Academy of Sciences

Abstract. For a map $f : M \to N$ between orientable manifolds of same dimension, a point $y \in N$ is called (*essentially*) *deficient* if $f^{-1}(y)$ has less then $|\deg f|$ (essential) points. Such points were studied by several topologist, e.g. Hopf, Honkapohja, Church, Timourian and Walsh.
The following result was proved by Church and Timourian.
Suppose M and N are connected orientable n-manifolds and $f : M \to N$ is a proper map with degree $|\deg f| \neq 0$. Let E_f be the set of essentially deficient points of f.

(1) *Then* $\dim E_f \leq n - 1$ *and, moreover,* E_f *contains no closed (in N) subset of dimension $n - 1$.*

(2) *If f is discrete, then* $\dim(\overline{E}_f) \leq n - 2$.

They also generalized this theorem to manifolds which are not necessarily orientable replacing $|\deg f|$ by Hopf's absolute degree $A(f)$.
We extend the notions of absolute degree and essentially deficient points with respect to maps from some spaces (in particular from CW-complexes) to manifolds. Then we prove a result, which extends the result of Church and Timourian, where domain is a space which satisfies certain cohomological property.
We consider also a related notion of *multiple* point of a map. A point $x \in X$ is a multiple point of a map $f : X \to Y$ if $f^{-1}(f(x)) \neq \{x\}$. We study density of multiple points in X, and also provide examples where its complement is dense.
This a joint work with Daciberg L. Goncalves and Thais F. M. Monis.

0-concordance of 2-knots

Nathan Sunukjian
Calvin College

Abstract. A 2-knot is defined to be an embedding of S^2 in S^4. Unlike the theory of concordance for knots in S^3, the theory of concordance of 2-knots is trivial. This talk will be framed around the related concept of 0-concordance of 2-knots. It has been conjectured that this is also a trivial theory, that every 2-knot is 0-concordant to every other 2-knot. We will show that this conjecture is false, and in fact there are infinitely many 0-concordance classes. We'll in particular point out how the concept of 0-concordance is related to understanding smooth structures on S^4. The proof will involve invariants coming from Heegaard-Floer homology, and we will furthermore see how these invariants can be used to shed light on other properties of 2-knots such as amphichirality and invertibility.

Gluing Formulas for Seiberg-Witten Invariants

Piotr Suwara
Massachusetts Institute of Technology

Abstract. One of the most important applications of Seiberg-Witten theory was the construction of families of exotic K3 surfaces via knot surgery carried out by Fintushel and Stern. This construction used gluing formulas obtained by Morgan-Mrowka-Szabo and Taubes. I would like to provide a short survey of how one can obtain gluing formulas using the tools of Monopole Homology.

Controlled Patterned Spaces

Mathew Timm
Bradley University

Abstract. Controlled pattern spaces are defined. They have interesting covering space properties. In particular, certain controlled patterned disks with holes can be used to construct spaces with interesting self covering properties.

Continued Fractions, Non-orientable Surfaces, and Torus Knots

Cornelia Van Cott
University of San Francisco

Abstract. We construct non-orientable surfaces which are bounded by torus knots. Using tools from knot Floer homology, we consider whether these surfaces realize the non-orientable four-ball genus of the knots. This is joint work with Slaven Jabuka.