The Little Logic Book
Hardy, Ratzsch, Konyndyk De Young and Mellema
The Calvin College Press, 2013

Exercises for The Little Logic Book may be downloaded by the instructor as Word documents and then modified for distribution to students; or students may be instructed to download the exercises and then told which ones to answer. This is an exercise bank; it is not assumed that students will do all the exercises for any one chapter. Comments, questions or suggestions for Chapter Three of The Little Logic Book can be sent to logic@calvin.edu

Chapter Three: Quantification

(Posted January, 2014)

1.0 Basic Concepts

Define or identify the following:

1.1 Universal quantifier
1.2 Existential quantifier
1.3 Predicate
1.4 Logically equivalent

2.0 Symbols

Identify the symbols for:

2.1 Universal quantifier
2.2 Existential quantifier
2.3 Equivalence

3.0 Quantification Basics

Indicate whether the following statements are true or false. Explain your answers.
(These may take some thought – or even some help from your instructor.)

3.1 If it is *not* true that everything has property P, then it *is* true that *at least one* thing lacks property P.
3.2 If it is *not* true that everything has property P, then it is true that *nothing* has property P.
3.3 If nothing, including even your laptop, has property P, then everything has the property ~P.
3.4 If (∃x)Px, then also (∃x) ~Px
3.5 If you know that *some* things have property P, and you know of nothing that *lacks* property P, then it logically follows that *most* things have property P.

4.0 Quantification Rules

State each of the following rules, including any special conditions. In your own words *explain* each rule, including the reason for any special conditions.

4.1 Universal instantiation
4.2 Existential generalization

4.3 Universal generalization
4.4 Existential instantiation

5.0 Equivalences
Each of the following expresses a logical equivalence. For each, give an intuitive explanation of why the expressions on each side of the triple bar (≡) mean exactly the same thing.

5.1 $(\exists x)Px \equiv \sim(x) \sim Px$.
5.2 $(x)Px \equiv \sim(\exists x) \sim Px$
5.3 $\sim (x)Px \equiv (\exists x) \sim Px$
5.4 $(x) \sim Px \equiv \sim(\exists x) Px$

6.0 Quantification in Arguments
For each of the following: *name* the quantification rule(s) involved, *indicate* whether the argument is valid or invalid, *explain* your answer, where appropriate discussing any special conditions.

6.1 All brown calironni mushrooms contain bisocarubyn, so the calironni mushrooms Melissa ate contain bisocarubyn.

6.2 Melissa ate some brown calironni mushrooms and the bisocarubyn they contain made her ill. From that we can conclude that everyone who eats brown calironni mushrooms will become ill.

6.3 Bisocarubyn always makes Melissa ill. All brown calironni mushrooms contain bisocarubyn, so any and all brown calironni mushrooms Melissa eats will result in her becoming ill.

6.4 Melissa ate some brown calironni mushrooms and the bisocarubyn they contain made her ill. From that we can conclude that at least some people who eat brown calironni mushrooms become ill.

6.5 All humans metabolize and react bisocarubyn in exactly the same way. Melissa ate some brown calironni mushrooms and the bisocarubyn they contain made her ill. From all that we can conclude that everyone who eats brown calironni mushrooms will become ill.

7.0 Showing Invalidity by Analogy
Construct your own analogous argument demonstrating that the following argument is invalid.

7.1 Porpoises and Mammals

1. Some porpoises are capable of diving to depths of over 400 feet.
2. Some mammals are not capable of diving to depths of over 400 feet.

Therefore,

3. Some mammals are not porpoises.

7.2 Rattlesnakes and Snakes

1. All rattlesnakes are poisonous.

2. Some snakes are not rattlesnakes

Therefore,

3. Some snakes are not poisonous.

8.0 A Stretch

8.1 Quantification is used in some important philosophical arguments. Below is a representation of an argument René Descartes gives in the first meditation of his *Meditations on First Philosophy* (1641), often regarded as the founding text of modern philosophy. In this work Descartes proposes to use doubt rather than the traditional method of dialectic (critical conversation) to get to the first principles from which an entire system of philosophical knowledge could be deduced. Those principles, in his view, had to be absolutely certain. So he proposed to use a particularly strong form of doubt, "hyperbolic doubt," in order to clear away any beliefs that were less than absolutely certain. If any beliefs were left standing after the exercise of hyperbolic doubt, they could serve as the foundation of this new system of philosophical knowledge.

He begins by considering all the beliefs he has acquired on the basis of direct sense perception. What could be more certain? But, he admits, it is at least possible that those beliefs have been delivered to him in a dream state, and thus could be false. So he must doubt them all. Next he considers beliefs that appear necessarily true to him when he thinks about them carefully, the so-called "truths of reason" (like a+b=b+a). What could be more certain than these objects of pure rational insight? But, it seems, it is at least possible that his mind was created by an "evil genius" such that what appeared to him to be necessarily true was in fact false. The evil genius finds this amusing, perhaps. So he must doubt these beliefs as well.

Here is the argument that leads him to reject the truths of reason as candidates for first principles of the system he wants to build:

1. Every belief I have based on rational insight might be a belief resulting from the deception of an evil genius.
2. Every belief that might be resulting from the deception of an evil genius is a belief that falls to hyperbolic doubt.
3. Every belief that falls to hyperbolic doubt is not absolutely certain.
4. Every belief that is not absolutely certain is a belief I should reject for philosophical purposes.

Therefore,

5. Every belief I have based on rational insight is a belief I should reject for philosophical purposes.

Analyze the inferences involved in this chain of philosophical reasoning. It might help to remember that logicians read statements like "All S is P" (or "Every S is P") as a conditional statement: "For all x, if x is an S, then x is P." The first premise of the argument above, then, could be construed in this way: "For all x, if x is a belief based on rational insight, then x might be a belief resulting from the deception of an evil genius." Or: $(x)Rx \rightarrow Dx$, where "R" stands for "Belief based on rational insight" and "D" stands for "Belief that might be resulting from the deception of an evil genius." Once universally quantified statements like these are put in the form of conditional statements and instantiated (UI), the arguments that employ such statements can be evaluated using the rules of deductive inference between propositions (see Chapter 1). At the end of the argument you can re-generalize the conclusion (UG) to get the universally quantified form of the statement. So, you can get started by taking $(x)Rx \rightarrow Dx$ to "If the belief a+b=b+a is based on a rational insight, then the belief a+b=b+a might be resulting from the deception of an evil genius" through UI and go on from there, repeating the same step for the other premises. At the end you can re-introduce a universal claim about all beliefs based on rational insight *if* the condition has been met for UG (that is, *if* the belief a+b=b+a was selected arbitrarily, if what is true of a+b=b+a in this case is true of all beliefs based on rational insight). After this analysis, if you decide that the reasoning in Descartes's argument is valid, are there any premises that strike you as false, or at least as questionable?